

光學量測聯合有限元素法對結構應力  
分析與損壞預測之研究  
**HYBRID-NUMERICAL METHOD ON STRESS  
ANALYSIS AND FAILURE PREDICTION OF  
DAMAGED STRUCTURES**

Jim Yang(楊玉井), Steven McNeill, Samuel Russell and  
Allen Nettles

原著載於中華民國第二屆結構工程研討會暨  
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楊玉井<sup>1</sup> Steven McNeill<sup>2</sup> Samuel Russell<sup>3</sup> Allen Nettles<sup>4</sup>

## ABSTRACT

Structural component of civil and other engineering are often inherited defects during manufacture or damages during construction and services. It is critical to understand the effect of damages and defects to the behavior of structure before its further use. To evaluate this effect and predict its failure, stresses around the irregular area must be closely estimated. A Hybrid-Numerical method which combines experimental technique and finite element modelling has been employed to analyze the stresses. First the displacement field around an irregular area is surveyed with Digital Image Correlation method. It is then used as boundary conditions in the finite element analysis. Structures made of composite materials with circular hole and impact damages have been investigated as examples in this report.

## INTRODUCTION

Structural component of civil and other engineering are often inherited defects during manufacture, such as voids, or damages during construction and services, such as cracks, spall and scabbing. It is critical to know the effect of defects and damages to the behavior of structure before its further use. To evaluate this effect and predict its failure, stresses around the irregular area must be closely estimated.

To understand the behavior and limitation of these structural components researchers usually use experimental tests or theoretical analyses on structures with simulated flaws. However, neither approach has been completely successful. As Durelli states that "Seldom does one method give a complete solution, with the most efficiency." [1] Examples of this principle is seen in photomechanics which additional strain-gage testing can only average stresses at locations of high concentration. On the other hand, theoretical analyses including numerical analyses are implemented with simplified assumptions which may not reflect actual boundary conditions.

Hybrid-Numerical methods which combine photomechanics and numerical analysis have been used to correct this inefficiency since 1950's. But its application is limited until 1970's when modern computer codes became available. In recent years, researchers [2-5] have enhanced the data obtained from photoelasticity, laser speckle, holography and Moire' interferometry for input of finite element analysis on metals. Nevertheless, there is only few of literature being done on composite laminates.

It is known the stress or strain at the singular point can not be accurately measured by an experimental technique. Nevertheless if the location is far away from singular spot, the displacement can be found locally regardless the boundary conditions, it is an excellent input data for a finite element analysis to replace the usually assumed boundary conditions. Therefore, the Hybrid-Numerical method is chosen to avoid the difficulty and to take advantage of both experimental technique and finite element analysis.

In this study, experimentally, the digital image correlation technique [6-8] is employed to measure the displacement field. It is done by comparing two digitized images, before and after loading. Numerically, the finite element program, ABAQUS (version 5.2) [9], is used to analyze the stress and strain field. It takes advantage of the high speed and huge memory size of modern supercomputer, CRAY Y-MP, at NASA Marshall Space Flight Center.

## DIGITAL IMAGE CORRELATION

Digital image correlation is based on the comparison between two digital images. As shown in the Figure 1, the system uses a standard Charge Couple Device (CCD) video camera attached to

<sup>1</sup>亞新工程顧問公司結構部計畫經理

<sup>2</sup>Associate Professor, University of South Carolina

<sup>3</sup>Senior Scientist, Marshall Space Flight Center, NASA

<sup>4</sup>Senior Engineer, Marshall Space Flight Center, NASA

video digitizer card to acquire digital images. The digitizer transforms an image to a 512x512 set of numbers representing the image. Each number represents the intensity of light impinging on a small area of camera sensor, which is called a pixel. The value of each pixel ranges from 0 to 255 with the lowest value representing black, highest value representing white, and values in between representing different shades of gray. An image processing software in a personal computer is then used to compare subsets of numbers between the two digital images.

As it can be seen in the Figure 2, two sets of image number (before and after a movement) are different because the object moves parallel to the camera's sensor plane. To measure how well the subsets match, a correlation function is used. By minimizing the correlation factor, the values of displacement and strain at any location of image can then be determined.

## FINITE ELEMENT ANALYSIS

Finite element analysis for stress/strain of a structure is based on the following equations of equilibrium:

$$[K] \{q\} = \{F\} \dots\dots\dots (1)$$

It is resulted by minimizing the potential energy of the whole structure. Where  $\{q\}$ ,  $\{F\}$ , and  $[K]$  represent nodal deformation, nodal loads, and structural stiffness matrices, respectively. Each member in  $\{q\}$  matrix is a degree of freedom. It is corresponding to a nodal force or moment in the same direction. For the static linear elastic problem, a degree of freedom is either unknown or known by fact or assumption. In the Hybrid-Numerical approach, some parts of  $\{q\}$  matrix will be filled with the displacements measured by the digital image correlation besides the regular assumed boundary conditions. Providing the stiffness matrix of structures,  $[K]$ , the unknowns in both  $\{q\}$  and  $\{F\}$  can be solved though a matrix condensation in a structural analysis program with a high speed computer.

The stiffness matrix of a structure,  $[K]$ , is assembled from the stiffness matrices of element. Each member of  $[K]$  matrix relates a degree of freedom to an associated nodal force or moment. The value of each member is determined by the geometry and the material properties of associated elements. Since each layer,  $[Q]$ , must be first formed in the structural coordinates system, or loading directions. And the loading-displacement relations is then constructed as the following form[10]:

$$\begin{bmatrix} [A] & [B] \\ [B] & [D] \end{bmatrix} \begin{bmatrix} \{\epsilon^0\} \\ [k] \end{bmatrix} = \begin{bmatrix} [N] \\ [M] \end{bmatrix} \dots\dots\dots (2)$$

Where  $[A]$ ,  $[B]$ , and  $[D]$  are determined by integrating the stiffness of all layers. Using above equations as the constitute equations of thin shell elements, the stiffness matrix of elements made of composite laminate can be formed.

This stiffness matrix of elements can be different depending on the material properties of individual element. In this study, a degraded material has been assumed to the damage areas. The elastic constants related to the transversal direction of a degraded lamina is assumed to be decreased by a degradation factor. Using these constants, the loading-displacement relations of damaged lamina can be found, and hence the stiffness matrix of damaged elements.

## FAILURE ANALYSIS

Failure is measured with failure criteria which depend on materials. Because composite materials is interested in this study, the failure modes and failing procedure of a laminates must be understood. And then a criterion can be chosen to predict the failure of composite laminates. As it is known that a lamina can be degraded and failed under following modes: (a) matrix failure with cracks; (b) fibers failure by being broken, buckled, or kinked as a group; (c) debonding between fiber and matrix; (d) delamination between layers. These failure modes interact and occur simultaneously as well as sequentially.

With above understanding researchers have proposed some criteria to assess the failure of composite laminates. Among them, the Tsai-Wu Theory[11] is mostly adopted for a polymer

composite lamina. According to this theory a lamina will have initial crack and start to degrade if its stress state can not satisfy the following inequality:

$$F_{ij} \sigma_i \sigma_j + F_i \sigma_i < 1 \quad \dots\dots\dots (3)$$

It works nicely when delamination and buckling are not concerned. Since linear elasticity has been assumed the Strength Ratio (R), ratio of stresses at failure to the tested stresses, can be estimated by the following equation:

$$(F_{ij} \sigma_i \sigma_j) R^2 + (F_i \sigma_i) R = 1 \quad \dots\dots\dots (4)$$

This Strength Ratio can be interpreted as how many times of current loading the lamina can be subjected before it starts to degrade. If every laminae are degraded the loading level is referred as Last Ply Failure of laminates. In Experiments acoustic events may be heard at this moment.

### EXAMPLES AND RESULTS

This Hybrid-Numerical method can be applied to concrete, steel and aluminum materials though, the examples of composite laminate structures are used to demonstrated in this report, because the generality of constitutive relation and the failure criterion. Two examples are presented as follows:

First, as shown in the Figure 3, specimens of quasi-isotropic,  $[0/45/-45/90]_S$  laminates made of T300/f934 prepregs with 1/8 inch diameter central circular hole are tested with uniaxial tension. Images are taken during the test for deformation evaluation. The area images is about 0.77 inch long and 0.91 inch wide. As it can be seen in the Figure 4, the maximum displacement at the edge of hole, which is measured by image correlation technique, is about twice of the value predicted by conventional finite element method. The values of displacement along boundary are staggering due to the inevitable error in the process. So that displacements along the boundary of this area are smoothed with the Least Square method before input in the stress analysis program. A computer code, ABAQUS, has been employed for this stress analysis[9].

An eight-nodes quadrilateral shell element is used for the finite element analysis. The edge of hole is divided into 128 elements to catch the stress concentration. Stresses at concentrate locations is then used to estimate the loading level of first lamina failure and Last Ply Failure. The result predicts that the specimen is totally degraded and its fibers may start to break when loading reaches 980 pound testing force. It agrees very well with the observation in the test while acoustic events has been clearly heard at about 950 pound testing force level.

In the second example, the cylindrical rocket motor cases are investigated. They are cylindrical pressure vessels made of IM7/Epoxy with the winding layout of  $[78.5/-78.5/0/0]_2$  from inside out. In which the 0 degree is referred to the circumferential direction. It is about 5.75 inch in diameter and 4 inch long (does not count both semispherical dome at ends). Every bottle has been subjected to a low speed impact test. They are three different impact energy levels, 3, 5 and 7 foot-pound applied at the middle of bottle and perpendicular to the composite laminate skin. The size of damage areas has been measured with shearography technique. It has been seen the smallest damage is scattered within 1"x1" area; and the largest is 3"x3". Based on the identified pattern of damage, the associated elements in the finite element analysis are assigned the degraded material group.

During the burst test of each pressure vessel, two images has been taken, one at free load and the other at 1000 psi pressure level. The calculation of digital image correlation runs over about 300 by 300 pixels. It covers an area of composite laminate about 1.90" by 1.61". The resulting displacements are then input as boundary in the finite element analysis. A mesh diagram with 20 by 20 rectangular thin shell elements is constructed. Using a computer code, ABAQUS, the stresses and strains of shell elements are calculated. And the stresses is then checked with the Tsai-Wu Theory to predict the pressure level at the Last Ply Failure of cylindrical bottle skin. Some typical results are shown in Table 1, it agrees with of the acoustic measurement, and it is much lower than that predicted by conventional finite element analysis, as reader can easily confirmed.

## CONCLUSION

The results of above examples has demonstrated that the Hybrid-Numerical Methods can closely predict the failure of composite structural components. We believe that it can be extended to concrete and steel structures without any difficulty. Since the portability of image-taken system and the advent of high capability personal computer, the Image Correlation Technique becomes very handy to do the site measurement of Civil structures, such as bridges and buildings. Furthermore, when it is integrated with a finite element analysis program, the Hybrid-Numerical method will be very practical and accurate enough for the design purpose in structural repair work.

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### 光學量測聯合有限元素法對結構應力分析與損壞預測之研究

楊玉井<sup>1</sup> Steven McNeill<sup>2</sup> Samuel Russell<sup>3</sup> Allen Nettles<sup>4</sup>

土木及其他結構物常在建造中存留缺陷或在施工及使用中受損。瞭解這些損害或缺陷對結構的影響是此結構能否繼續使用的要務。然而要理解它對結構的影響，並預測其損壞，就必須先對受損或缺陷區的應力加以精確的評估。本文介紹一種光學量測結合有限元素法分析此異常區的應力。此法利用實驗力學常用之光學量測技術估量異常區內之位移，然後以此位移為有限元素分析之邊界條件計算區內應力。為證明此法之效用，特舉複合材料之板開洞及高壓火藥筒受撞擊為例。

Table 1. Burst Pressure of Cylindrical Rocket Bottle Cases

Serial #	Impact Level	Predicted by Hybrid-Numerical Method	Test Result
F.E.M.	0	4210 psi	N/A
A-021-02	3 ft-lb	2760 psi	2616 psi
C-081-082	5 ft-lb	2620 psi	2512 psi
A-001-002	7 ft-lb	2470 psi	1800 psi*

\* Fiber breaking due to impact has been detected before burst test.

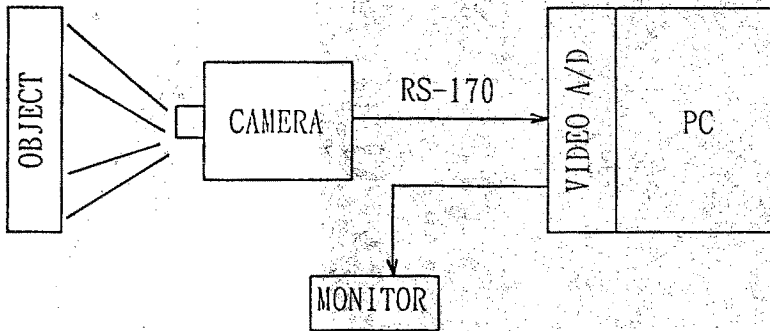
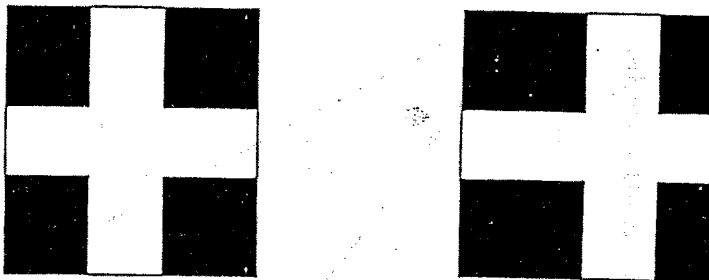


Fig.1- Schematic diagram of digital-image-processing



0	0	255	255	0	0	0	0	0	255	255	0
0	0	255	255	0	0	0	0	0	255	255	0
255	255	255	255	255	255	255	255	255	255	255	255
255	255	255	255	255	255	255	255	255	255	255	255
0	0	255	255	0	0	0	0	0	255	255	0
0	0	255	255	0	0	0	0	0	255	255	0

Fig.2 Image numbers of object

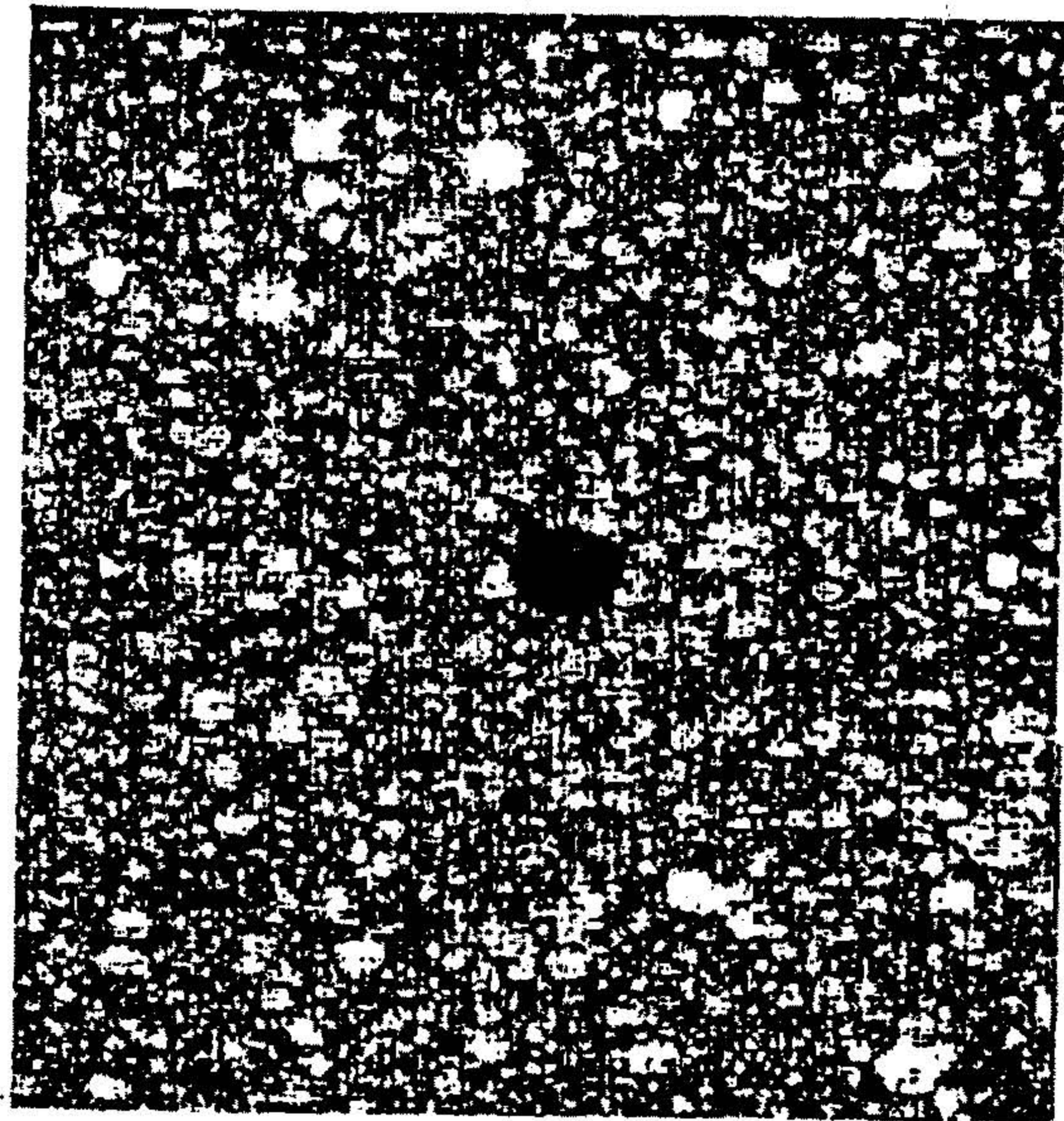
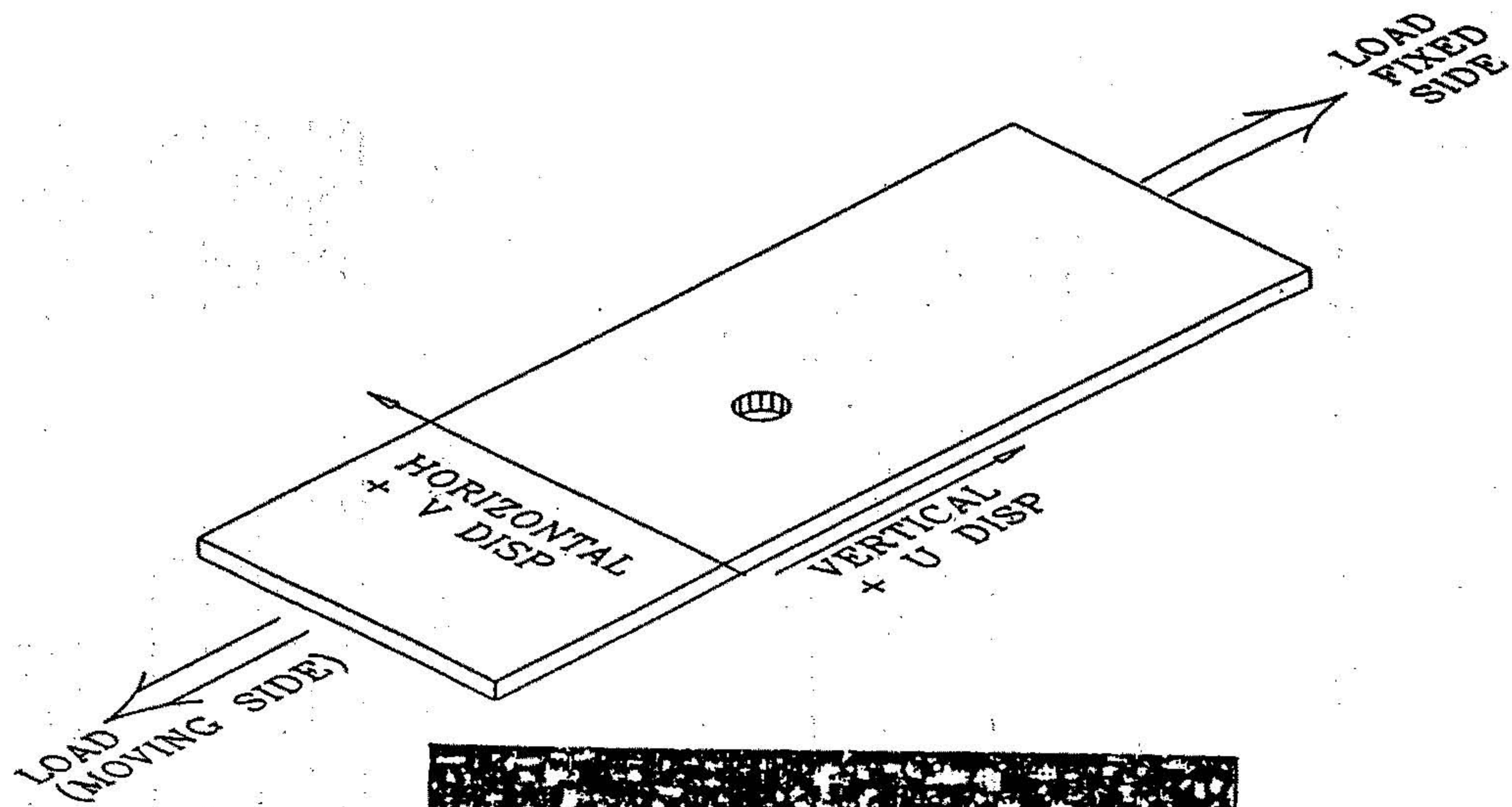


Fig.3 Composite Laminate with a Circular Hole

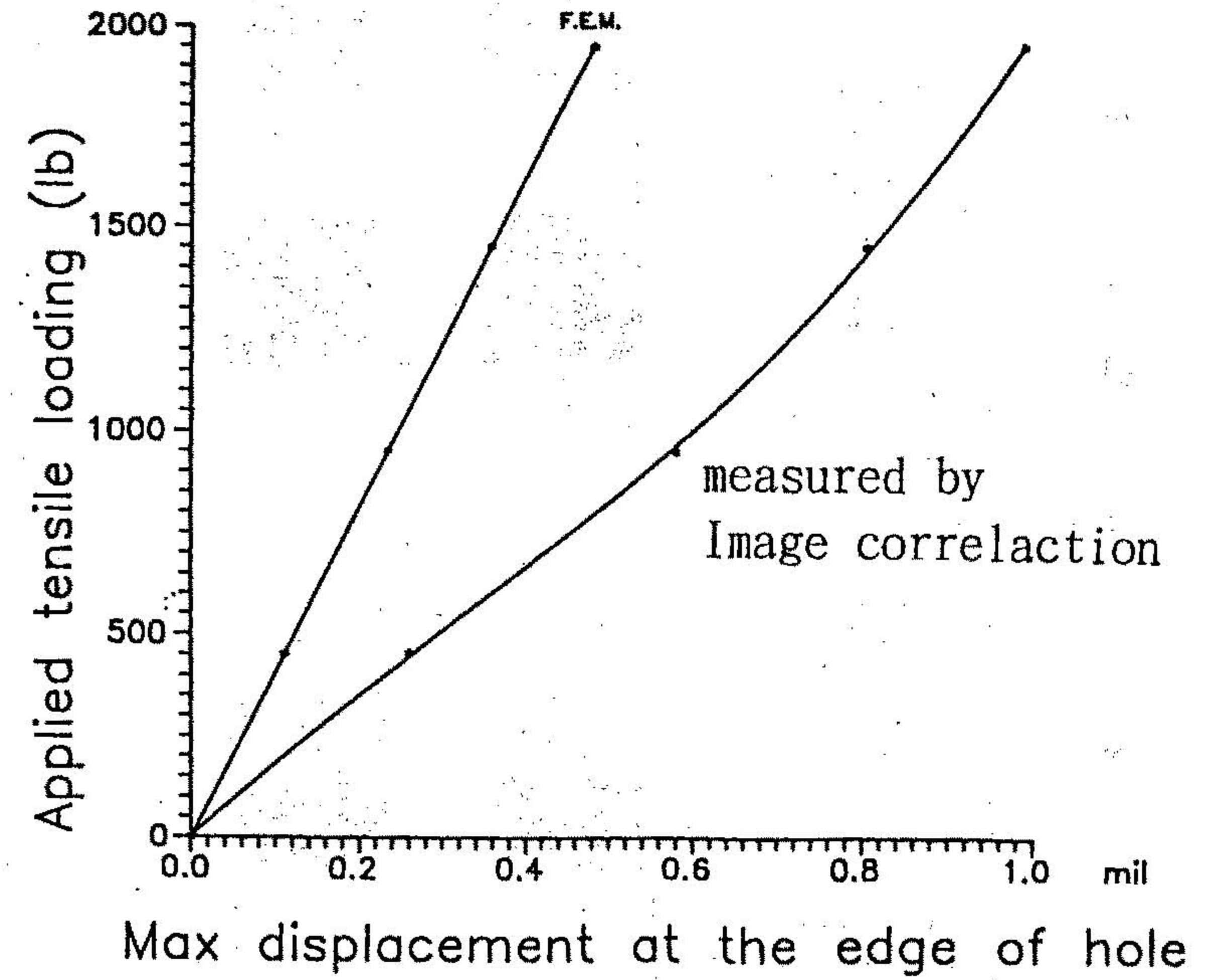


Fig.4 Max. displacement at the edge of hole